Representative radial velocity profiles are presented in Fig. 3. In general V_r is seen to increase with increasing z in the neighbourhood of the wall, attain a maximum, and then to decay asymptotically to zero at large z. The effect of slip is to diminish somewhat the magnitude of the velocity maximum and to shift its location closer to the wall.

ACKNOWLEDGEMENTS

This work arose out of research programs supported by the National Science Foundation under Grant GK-13303 and by the Department of Defense under Project THEMIS, N00014-68-A-0141-0001, administered by the Office of Naval Research. The support of these agencies is gratefully acknowledged.

REFERENCES

1. G. S. BEAVERS and D. D. JOSEPH, Boundary conditions at a naturally permeable wall, *J. Fluid Mech.* **30**, 197–207 (1967).
2. G. S. BEAVERS, E. M. SPARROW and R. A. MAGNUSON, Experiments on coupled parallel flows in a channel and a bounding porous medium, *J. Basic Engng* **92**, 873–878 (1970).
3. H. SCHLICHTING, *Boundary-Layer Theory*, 6th ed., translated by J. KESTIN. McGraw-Hill, New York (1968).

Int. J. Heat Mass Transfer. Vol 14, pp. 996–998. Pergamon Press 1971. Printed in Great Britain

A DOUBLE-RAY TECHNIQUE FOR THE INVESTIGATION OF LIQUID BOUNDARY LAYERS

E. A. SCHWANBOM, D. BRAUN, E. HAMANN and J. W. HIBY
Institut für Verfahrenstechnik, Technische Hochschule, Aachen, Germany

(*Received* 28 *July* 1970)

1. INTRODUCTION

FOR THE experimental verification of mass transfer theories, the measurement of the local and time-dependent concentration distribution within the boundary layer appears to be essential. Probe techniques, however, are for this purpose limited to measurements in the gaseous phase. For the very thin liquid boundary layers, only optical methods seem applicable. Indeed, interference methods have been adapted by Lin *et al.* [1] to a solid–liquid interface and by Jepsen *et al.* [2] to a gas–liquid interface. A great disadvantage of the interference method is that the direction of observation must be parallel to the boundary surface, and therefore only an averaged value along this direction is obtainable. Furthermore the boundary surface must be plane.

2. THEORETICAL BASIS AND DESCRIPTION OF THE METHOD

The use of a pH-indicator offers another possibility of measuring the concentration distribution. This method has been adapted to the case of mass transfer from the gas phase to a falling film [3, 4]. Here the direction of observation is perpendicular to the interface and the resolution in directions along and perpendicular to the interface is very high. The time average of the local concentration distribution was evaluated and from this the spatial distribution of the effective diffusivity. This gives a detailed picture of turbulent mass transfer.

Mass transport is obviously interrelated with the wave formation in the falling film. This calls for an extension of the pH-indicator method which would allow to recognize the correlation between wave dynamics and transport mechanism. A solution to this problem was found in a double-ray technique which was originally developed for the measurement of concentration profiles near a solid wall. The method was adapted to the falling film by T. Melin [5].

The new method makes use of the fact that many colour indicators—as well as a few fluorescent indicators—exhibit one or more so-called isosbestic (isoemissive) points (Fig. 1). When gaseous ammonia is absorbed in a falling film of a diluted strong acid, containing a minute amount (about 10^{-5} moles/l) of the fluorescent indicator acridine, two distinct regions are formed (Fig. 2): at the surface an

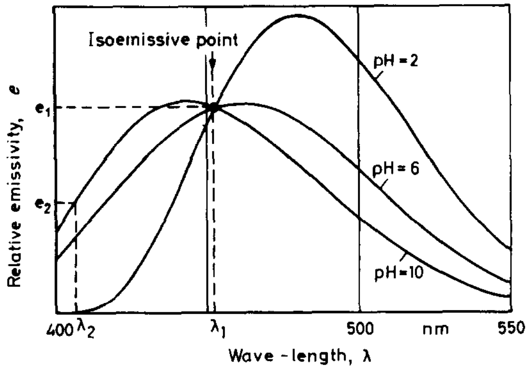

FIG. 1. Emission spectra of acridine (10^{-5} moles/l) stimulated by ultraviolet radiation (365 nm). At the isoemissive wavelength the emissivity is independent of the pH-value of the film.

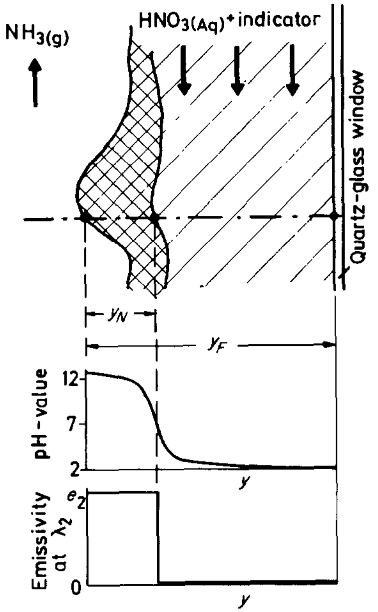

FIG. 2. Absorption and reaction of NH_3 in falling film.

alkaline region of depth y_N, where ammonia is in excess, and an acidic region comprising the rest of the total film thickness y_F, where non-reacted acid still exists. The smaller the concentration of the entering acid, the greater the value of y_N. It has been shown that this offers a possibility to evaluate the concentration profile [3, 4].

When the film is exposed to ultraviolet radiation, the acridine indicator emits light with a wave-length distribution according to Fig. 1. The emitted light is collected by a lense and divided into two parts, each one passing through an interference filter which transmits a narrow band at the wave-lengths λ_1 and λ_2 respectively, and thereafter focused upon a photomultiplier (Fig. 3). λ_1 is chosen at the isoemissive point, whereas at the wave-length λ_2 practically only the alkaline region of the liquid emits light. (The small contribution of the acid emissivity can be compensated against the λ_1-intensity in a bridge circuit.)

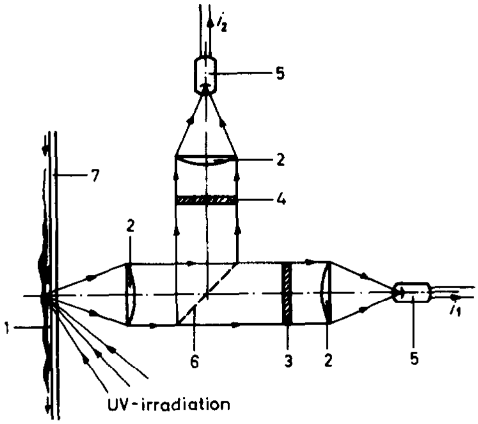

FIG. 3. Optical apparatus: (1) Liquid falling film, (2) lenses, (3) interference filter λ_1, (4) interference filter λ_2, (5) photomultiplier, (6) semitransparent mirror and (7) quartz-glass window.

For the anode currents i_1 and i_2 of the two multipliers we have

$$i_1 = A_1 . e_1 . y_F,$$

$$i_2 = A_2 . e_2 . y_N.$$

Therefore the signal at the isoemissive wave-length λ_1 gives the instantaneous and local value of the film thickness y_F, whereas the signal at the wave-length λ_2 gives at the same time and on the same spot of the film the distance y_N between the phase boundary and the neutralisation plane within the film. The constant factors $A_1 . e_1$ and $A_2 . e_2$ are determined by calibration in a cuvette of known thickness.

REFERENCES

1. C. S. LIN, R. W. MOULTON and G. L. PUTNAM, Interferometric measurements of concentration profiles in turbulent and streamline flow, *Ind. Engng Chem.* **45**, 640–646 (1953).
2. J. C. JEPSEN, O. K. CROSSER and R. H. PERRY, The effect of wave induced turbulence on the rate of absorption of gases in falling liquid films, *A.I.Ch.E. Jl* **12**, 186–192 (1966)

3. J. W. HIBY, Eine Fluoreszenzmethode zur Untersuchung des Transportmechanismus bei der Gasabsorption im Rieselfilm, *Wärme- und Stoffübertragung* **1**, 105–116 (1968).
4. J. W. HIBY, D. BRAUN and K. H. EICKEL, Eine Fluoreszenzmethode zur Untersuchung des Stoffüberganges bei der Gasabsorption im Rieselfilm, *Chemie-Ingr.-Tech.* **39**, 297–301 (1967).
5. T. MELIN, Diploma thesis, Institut für Verfahrenstechnik, Aachen (1970).

Int. J. Heat Mass Transfer. Vol. 14, pp. 998–1001. Pergamon Press 1971. Printed in Great Britain

THE TEMPERATURE DISTRIBUTION IN AN INFINITE MEDIUM RESULTING FROM A LINE SOURCE OF FINITE DURATION*

ANTONIO MONTEALEGRE and HERBERT WEINSTEIN

Department of Chemical Engineering, Illinois Institute of Technology, Chicago, Illinois, U.S.A.

(*Received* 16 *June* 1970 *and in revised form* 7 *December* 1970)

NOMENCLATURE

c,	heat capacity;
C_n,	constant;
$g(r)$,	temperature distribution function at time t_0;
k,	thermal conductivity;
M_n,	solution function;
p,	period of heat generation;
Q',	amount of energy generated instantaneously per unit length;
Q,	amount of heat generated per unit length per unit time;
r,	radial position;
t,	time;
T,	temperature;
ρ,	density;
η,	transformation variable.

INTRODUCTION

THE PROBLEM treated in this work is that of finding the temperature distribution in an infinite medium during and following the generation of energy over a finite time period by an infinite line source. If the solution to a unit instantaneous line source is known and integrable, the temperature distribution resulting from a line source can readily be found by integration. The solution to a unit instantaneous line source is given in Carslaw and Jaeger [1] as obtained by Green's functions. It is a simple expression but is not always easily integrable, especially if the heat generation rate varies with time

SOLUTION TO LINE SOURCES AND SINKS IN AN INFINITE MEDIUM

The energy equation for the region around an instantaneous line source generated at time $t = t'$ can be written as

$$\rho c \frac{\partial T}{\partial t} = \frac{k}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right), \quad t > t' \tag{1}$$

with boundary conditions

$$T(t, \infty) = 0 \tag{2a}$$

$$\frac{\partial T}{\partial r}(t, o) = 0 \tag{2b}$$

and global conservation of energy expressed as

$$2\pi \int_0^\infty \rho c T r \mathrm{d}r = Q' \tag{2c}$$

where Q' is defined as the strength of the source.

The solution to the above equations can be given in the form of a series (2)

$$T(t, r) = \sum_{n=0}^{\infty} \frac{C_n M_n \exp(-\eta^2/2)}{t^{n+1} \eta} \tag{3}$$

---

* The full support of this work by NASA under Grant No. NGR 14-004-008 (041) from the Space Nuclear Propulsion Office is gratefully acknowledged.